Relevance of the strange quark sector in chiral perturbation theory¹

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Abstract

Results obtained in recent years in the strange quark sector of chiral perturbation theory are reviewed and the theoretical relevance of this sector for probing the phase structure of QCD at zero temperature with respect to the variation of the number of massless quarks is emphasized.

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1 Chiral perturbation theory

Chiral perturbation theory (ChPT) is an effective theory of QCD at low energies, in which the dynamical degrees of freedom are those of the pseudoscalar Goldstone bosons (π , K, η) of the chiral group $SU(3) \times SU(3)$. That theory was developed by Weinberg [1] and applied in more detail to the QCD case by Gasser and Leutwyler [2, 3]. The effective lagrangian \mathcal{L}_{eff} is the most general locally chiral invariant lagrangian in the presence of external source terms. It contains an infinite series of terms constructed out of the pseudoscalar meson fields and the external sources. The series is arranged according to an increasing power of derivatives and quark masses. At low energies, the Goldstone boson interactions are weak (they are of second order in the derivatives and of first order in the quark masses) and therefore a perturbative calculation of their transition amplitudes is meaningful. The expansion parameter is essentially the order of magnitude of the external momenta or of the quark masses divided by the hadronic mass scale (which is of the order of 1 GeV).

The counting rules of the dimensionalities of various diagrams are based on the counting of the numbers of external momenta, of the mass terms and of loops, each quark mass being equivalent to a momentum to the power 2. Furthermore, because of the weakness of the interactions at the tree level at low momenta, each loop introduces two additional powers of momenta, thus contributing to nonleading orders [1]. Generally, there are terms of order $O(p^2)$, then terms of order $O(p^4)$, $O(p^6)$, etc. The effective lagrangian takes the following corresponding expansion:

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots , \qquad (1)$$

each index indicating the power of the momenta that will emerge from a tree-level calculation of a process. It is evident that at low energies it is the terms with smallest indices that will be the dominant ones.

The effective lagrangian is renormalizable order by order in perturbation theory. The term \mathcal{L}_2 contributes, through its one-loop effects, to the $O(p^4)$ terms, which are generally ultra-violet divergent. Those divergences are then absorbed in redefinitions of the coupling constants contained in the terms of the lagrangian \mathcal{L}_4 . The latter, in turn, produces, with \mathcal{L}_2 , one-loop divergences of order $O(p^6)$ that are absorbed, together with the two-loop divergences of the lagrangian \mathcal{L}_2 , by the coupling constants of the lagrangian \mathcal{L}_6 , and so forth.

At each order of the perturbation series there are a certain number of coupling constants, called low energy constants (LEC), that are order parameters of spontaneous chiral symmetry breaking. They are expected to be determined from experimental data. In addition, one also encounters the quark masses m_u , m_d , m_s .

At order $O(p^2)$, one has two LECs: F_0 , which is the pion weak decay constant F_{π} in the chiral limit, and $B_0 = -\langle 0|\overline{u}u|0\rangle_0/F_0^2$, which is proportional to the quark condensate in the vacuum in the chiral limit [3, 4]. At order $O(p^4)$, one has 10 (observable) LECs, called L_i , $i = 1, \ldots, 10$ [3]. At order $O(p^6)$, one has 90 (observable) LECs, called C_i , $i = 1, \ldots, 90$ [5]. It is worthwhile to emphasize that not all LECs enter in a definite process.

The theory relatively simplifies if one sticks to processes related to the nonstarnge sector of the quarks (u, d). The chiral group now becomes $SU(2) \times SU(2)$. Here, the strange quark can be considered as heavy and the corresponding field integrated out.

The $SU(2) \times SU(2)$ version of ChPT contains less LECs than its $SU(3) \times SU(3)$ version. At order $O(p^2)$ one still has two LECs, F and B, the analogs of F_0 and B_0 , but now considered in the $SU(2) \times SU(2)$ chiral limit. At order $O(p^4)$, there are 7 LECs, called ℓ_i , $i = 1, \ldots, 7$ [2]. At order $O(p^6)$, there are 53 LECs, called c_i , $i = 1, \ldots, 53$ [5].

A detailed study of the elastic $\pi\pi$ scattering amplitude up to order $O(p^6)$ was done by several groups [6, 7, 8, 9]. The rate of convergence of ChPT seems rather satisfactory: $O(p^4)$ effects represent approximately 20-25% of the global quantities under consideration, while $O(p^6)$ effects represent 7-8% of the contributions. Recent experimental data from the E865 experiment at Brookhaven about K_{e4} decay [10], which provides information about the $\pi\pi$ scattering amplitude near threshold through the final state interaction, have confirmed the hypothesis that the quark condensate parameter B is the leading order parameter of chiral symmetry breaking [4, 9, 11]. This means that the QCD vacuum is very similar to a ferromagnetic medium, as far as chiral symmetry breaking is concerned.

Extension of ChPT to $SU(3) \times SU(3)$ allows the study of sectors involving the K and η mesons. But here, the strange quark mass m_s is not as small as the non-strange ones, m_u and m_d . From the tree-level relation [4] $2m_s/(m_u+m_d)=2m_K^2/m_\pi^2-1\simeq 25$ one guesses that ChPT might converge more slowly than in the $SU(2)\times SU(2)$ case. Apart from that aspect, which by itself leads to unavoidable complications, it was emphasized by Descotes, Girlanda and Stern [12] that the type of dependence of physical quantities on the strange quark mass m_s might reveal some important theoretical features of QCD.

2 Phase transition in the number of massless quarks

It is known that $SU(N_c)$ gauge theories might undergo a zero temperature chiral phase transition when the number N_f of massless fermions (in the fundamental representation) reaches some critical value $N_f^* < 11N_c/2$ [13]. The argument goes as follows. For $N_f < 11N_c/2$, one has an asymptotically free theory, while for $N_f > 11N_c/2$ asymptotic freedom is lost. From the first two terms of the beta-function one infers the existence of an infra-red

stable fixed point appearing when N_f reaches from below a critical value $N_{f0}(<11N_c/2)$. In perturbation theory, one has $N_{f0} \simeq 34N_c/13$. When the value of N_f further increases and reaches the vicinity of $11N_c/2$ from below, then the domain of variation of the effective coupling constant becomes tiny and the theory becomes fully perturbative, in the infrared and in the ultra-violet, reducing to a conformal theory. Such theories, because of the smallness of the coupling constant, do not undergo spontaneous chiral symmetry breaking, neither display confinement [14]. On the other hand, at small values of N_f , one has chiral symmetry breaking and confinement [15, 16, 17, 18]. Therefore, there should exist a critical value of N_f , N_f^* , such that $N_{f0} < N_f^* < 11N_c/2$, where the theory undergoes a chiral phase transition (Fig. 1).

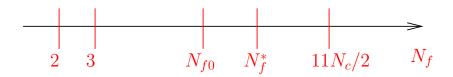


Figure 1: Domains of N_f . At N_{f0} , reached from below, an infra-red fixed point appears. At N_f^* a chiral phase transition from the Goldstone mode $(N_f < N_f^*)$ to the Wigner mode $(N_f^* < N_f < 11N_c/2)$ occurs. Above $11N_c/2$ asymptotic freedom is lost.

It is evident that at N_f^* all chiral order parameters should have vanished. Among those, F_0 would be the last one to vanish, since it is the fundamental order parameter of chiral symmetry breaking, directly related to the Goldstone theorem.

The precise value of N_f^* is dependent on the dynamical models that are used to evaluate it. Appelquist, Terning and Wijewardhana [14], combining perturbation theory and gap equation calculations find $N_f^* \simeq 4N_c = 12$. Lattice calculations give a rather wide range of values. Kogut and Sinclair [19], Brown et al. [20] find $8 \leq N_f^* \leq 12$; Iwasaki et al. [21] find $N_f^* \simeq 6$, while Mawhinney [22] finds $N_f^* \simeq 4$. Within the instanton liquid model, Velkovsky and Shuryak [23] find $N_f^* \simeq 6$.

From another viewpoint, Descotes, Girlanda and Stern have studied the dependence on N_f of various chiral order parameters [12]. Using properties of the Dirac operator in the background gluon field in euclidean space (placed in a box) and bounds derived by Vafa and Witten [24] concerning its eigenvalues, they obtain the following inequalities when the number of massless fermions changes from N_f to $N_f + 1$:

$$F_0^2[N_f+1] < F_0^2[N_f],$$
 (2)

$$|\langle \overline{u}u \rangle|_{(N_f+1)} \langle |\langle \overline{u}u \rangle|_{N_f}.$$
 (3)

These inequalities, which do not hinge on any hypothesis about the existence of a chiral

phase transition, are manifestly compatible, at least locally, with the behaviors of order parameters as expected from such an hypothesis.

In summary, the eventual existence of a chiral phase transition in N_f would have the tendancy to decrease the values of order parameters with increasing N_f . The slope of the variation would strongly depend on the value of the critical point N_f^* . It would be stronger for smaller N_f^* . In the real world one does not have much freedom to vary N_f . The only possibility that we have is to vary N_f from 2 $(SU(2) \times SU(2))$ to 3 $(SU(3) \times SU(3))$. (For $N_f = 1$, chiral symmetry is destroyed by the $U_A(1)$ -anomaly.) Possibly small values of N_f^* (4-6, say) would induce rather strong variations of order parameters in passing from $N_f = 2$ to $N_f = 3$. Therefore, phenomenological studies of such effects would represent indirect tests about the vicinity of N_f^* . In particular, quantities that are Zweigrule suppressed in the large- N_c limit (almost true for $N_f = 2$ and $N_c = 3$), should be enhanced for $N_f = 3$ [12]. Those mainly concern scalar meson sectors and the LECs L_4 and L_6 . Simultaneously, loops of the strange quark might provide important contributions and destabilize certain results obtained in the large- N_c limit [12].

3 Phenomenology

The above problem was first studied by Moussallam [25]. He calculated the ratio of the quark condensate evaluated in a theory with three massless quarks to the condensate evaluated in a theory with two massless quarks:

$$R_{32} \equiv \frac{\langle \overline{u}u \rangle_{N_f=3}}{\langle \overline{u}u \rangle_{N_f=2}}.$$
 (4)

In the $N_f = 2$ case, the mass of the strange quark is fixed at its "physical" value; but since the latter is still small compared to the massive hadron masses, one can use perturbation theory for it and keep only the leading contribution in m_s . One thus obtains, at the one-loop level:

$$R_{32} = 1 - \frac{m_s B_0}{F_{\tau}^2} \left[32L_6(\mu) - \frac{1}{16\pi^2} \left(\frac{11}{9} \ln(\frac{m_s B_0}{\mu^2}) + \frac{2}{9} \ln(\frac{4}{3}) \right) \right] + O(m_s^2), \tag{5}$$

where μ is the renormalization mass. The quantity $m_s B_0$ can be replaced by its tree-level expression, $(m_K^2 - m_\pi^2/2)$. L_6 is then evaluated from the correlator of scalar-isoscalar densities, $(\overline{u}u + \overline{d}d)$ and $\overline{s}s$:

$$\int d^4x e^{ip.x} \langle T[(\overline{u}u(x) + \overline{d}d(x))\overline{s}s(0)] \rangle_c.$$
 (6)

This is precisely a Zweig-rule violating term. It is evaluated by saturating the intermediate states with $\pi\pi$ and $K\overline{K}$ states, yielding the pion and kaon scalar form factors. One obtains

coupled Muskelishvili–Omnès equations. Use of experimental values of phase shifts and phases leads to:

$$L_6(m_n) = (0.6 \pm 0.2) \times 10^{-3},$$
 (7)

$$R_{32} = 0.46 \pm 0.27. \tag{8}$$

The last result indicates a strong variation of the quark condensate when passing from two massless quarks to three.

A similar study, by a different method, was also done in Ref. [26], confirming the above conclusions. $O(p^6)$ effects, estimated by means of a resonance model and the sigma-model, do not seem to qualitatively change the above results [27].

An important quantity in the strange quark sector is the πK elastic scattering amplitude. The tree-level (current algebra), $O(p^2)$, values of the S-wave isospin 1/2 and 3/2 scattering lengths had been calculated by Weinberg [28]:

$$a_0^{1/2} = 0.14, a_0^{3/2} = -0.07.$$
 (9)

(In units of m_{π}^{-1} .)

The scattering amplitude at the one-loop level, $O(p^4)$, was calculated by Bernard, Kaiser and Meissner [29]. The scattering lengths become:

$$a_0^{1/2} = 0.19, a_0^{3/2} = -0.05.$$
 (10)

Until recently, experimental knowledge of the scattering lengths was very poor. As for $\pi\pi$, it is not possible to realize direct scattering experiments at low energies, because pions and kaons decay. One must then use extrapolations of high energy data to low energies. Recently, a detailed evaluation of the low-energy πK elastic scattering amplitude was done by Büttiker, Descotes-Genon and Moussallam [30], by means of Roy and Steiner type equations [31, 32]. Those equations use dispersion relations, crossing symmetry, unitarity and partial-wave analysis, together with high-energy data, to reconstruct the elastic scattering amplitude at low energies. The method was already used for $\pi\pi$ scattering [33, 34, 35, 36, 9]. In πK , one ends up with six coupled integral equations. Solutions with rather small uncertainties have been obtained for the scattering lengths [30]:

$$a_0^{1/2} = 0.224 \pm 0.022, a_0^{3/2} = -0.045 \pm 0.008.$$
 (11)

 $O(p^6)$ effects in $\pi\pi$ and πK scattering were evaluated (for $N_f=3$) by Bijnens, Dhonte and Talavera [37, 38]. The corresponding LECs are calculated by resonance saturation methods. They do an overall fit to all existing data (scattering amplitudes, form factors, masses, etc.), leaving the Zweig-rule violating LECs L_4 and L_6 as free parameters. They

obtain several sets of results, depending on which experimental quantities optimization is imposed by varying slightly the resonance parameters. For the set producing the best fit with the scattering amplitudes, they find for the πK scattering lengths:

$$a_0^{1/2} = 0.220, a_0^{3/2} = -0.047.$$
 (12)

The relevant LECs are:

$$L_4 = 0.2 \times 10^{-3}, \qquad L_6 = 0.0 \times 10^{-3}.$$
 (13)

The above values of the scattering lengths match, within the allowed uncertainties, those obtained from the Roy-Steiner extrapolation method of high energy data [Eq. (11)]. The values of the LECs are also compatible with a small violation of the Zweig rule. At this point one might conclude that ChPT is rapidly converging, without sizable Zweig-rule violating effects. However, the overall fit of Refs. [37, 38] displays in some instances contradictory aspects. One notices that in other sectors (mainly the scalar ones) $O(p^6)$ effects are more important than $O(p^4)$ effects, indicating bad convergence (in particular in the pionic sector, which, in the $SU(2) \times SU(2)$ case had a rapid convergence). In such cases, the meaning of the $O(p^4)$ LECs L_4 and L_6 becomes questionable. Perhaps optimization with respect to global convergence of $SU(3) \times SU(3)$ should be tried.

In this respect, Descotes-Genon, Fuchs, Girlanda and Stern have proposed a different method of evaluation of high-order effects [39]. They suggest to isolate those terms which might be sensitive to Zweig-rule violating effects (four in all) and to treat them nonperturbatively, while treating the rest perturbatively. The method was already applied to the $\pi\pi$ scattering case; its application to the other sectors could still reduce the existing uncertainties.

Finally, we mention here some future useful experiments about the πK system.

The process

$$D \longrightarrow \pi K e \nu,$$
 (14)

could be analyzed in the FOCUS experiment at FermiLab. It would give information, through the final state interaction, about the elastic πK phase shifts. It plays an analogous role as the $K_{\ell 4}$ decay for $\pi \pi$ scattering.

The process

$$\tau \longrightarrow \pi K \nu,$$
 (15)

could be analyzed in the CLEOIII experiment at Cornell. It would give information about the $K_{\ell 3}$ form factors.

The observation and measurement of the properties of the hadronic atom $(K^+\pi^-)_{at.}$ would also give complementary informations about the scattering lengths. Hadronic atoms

are Coulomb bound states of charged hadrons, which generally decay under the effect of the strong interactions into neutral isospin partners. Thus the above atom would mainly decay as

$$(K^+\pi^-)_{at.} \longrightarrow K^0\pi^0. \tag{16}$$

The lifetime of the atom depends essentially on the combination $(a_0^{1/2} - a_0^{3/2})$ of the πK scattering lengths, while the energy level splittings depend upon the combination $(2a_0^{1/2} + a_0^{3/2})$ [40, 41, 42, 43]. The experimental study of the pionium (the $\pi^+\pi^-$ -atom) is currently done at CERN in the DIRAC experiment. For the $K^+\pi^-$ -atom, projects are being prepared. To precisely reconstruct the strong interaction scattering lengths from the hadronic atom properties, one needs to take into account isospin breaking and electromagnetic radiative corrections, as well as relativistic corrections. A recent theoretical study of the $K\pi$ -atom was done by Schweizer [44].

In conclusion, the study of the strange quark sector up to order $O(p^6)$ offers the possibility of a full test of $SU(3) \times SU(3)$ ChPT and at the same time of an indirect probe of a possible phase transition in the number of massless flavors in QCD.

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